

## Some practice

One common way to practice the art of linear approximations is to compute with explicit functions. On one hand it sounds silly since we actually know the function and could compute any value we would like, on the other it allows us to check what is going on and get some insights of the process.

### A typical exam question

Using a linear approximation, estimate the value of  $\sqrt{9.12}$

Note: during exams, you are not allowed calculators, so try to solve this problem without a calculator (you really won't need it).

linearize  $f(x) = \sqrt{x}$  at  $a = 9$

positive  
square  
root  
only  $\rightarrow \sqrt{9} = 3$

Solve  $x^2 =$

$x = \pm 3$   
 $x = \pm \sqrt{9}$

$$L_q(x) = f(q) + f'(q)(x - q)$$

$$L_9(x) = 3 + \frac{1}{6}(9.12 - 9)$$
$$= 3 + \frac{0.12}{6}$$

$$f(q) = \sqrt{9} = 3$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(q) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

$$\boxed{\sqrt{9.12} \approx 3.02}$$

(why not use  $a=1$ ,  $a=4$ ,  $a=16$ ?)  
 $a=9$  is the closest